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An a priori inequality concerning elliptic second order partial differential equations of variational type

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SUMMARY. — *I give an explicit evaluation of the constant appearing in an « a priori » inequality for the solutions of linear second order elliptic partial differential equations in divergence form. An application is also considered.*

1. *Introduction.*

About the theory of linear second order elliptic partial differential equations in divergence form with discontinuous coefficients, studied by G. Stampacchia in [3], the following maximum principle is known (see [1], [3]):

Suppose $u \in H^1(\Omega)$,

$$a(u, v) \equiv \int_\Omega \left\{ \sum_{i,j=1}^n a_{ij} u_{x_i} v_{x_j} + \sum_{i=1}^n (b_i u_{x_i} v + d_i u v_{x_i}) + cuv \right\} dx \leq 0$$

for any $v \in H_0^1(\Omega)$, $v \geq 0$ *in* Ω. *Under suitable hypotheses on the coefficients* a_{ij}, b_i, d_i, c $(i, j = 1, 2, \ldots, n)$ *it follows*

$$\operatorname*{ess\,sup}_\Omega u \leq \max(0, \max_{\partial\Omega} u).$$

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Given any $f_1, f_2, \dots, f_n \in L_2(\Omega)$, this result implies uniqueness and existence of the solution w of the Dirichlet problem

$$(1)\qquad \begin{cases} a(w, v) = \sum_{i=1}^{n} \int_{\Omega} f_i v_{x_i} \, dx & \forall\, v \in H_0^1(\Omega), \\ w \in H_0^1(\Omega). \end{cases}$$

From the closed graph theorem it follows also the existence of a constant K such that the solution w of (1) satisfies the inequality

$$\| w \|_{H^1(\Omega)} \leq K \sum_{i=1}^{n} \| f_i \|_{L_2(\Omega)} .$$

Nevertheless, as far as I know, the problem of finding the dependence of this constant K on the coefficients is open (except the trivial case when the bilinear form $a(\cdot, \cdot)$ is coercive on $H_0^1(\Omega)$, i. e. $a(z, z) \geq k \| z \|^2_{H^1(\Omega)}$ $\forall\, z \in H_0^1(\Omega)$, with k positive constant).

The aim of the present work is to give an upper bound of K through the ellipticity constant ν, the norms in $L_n(\Omega)$ of the functions $b_i - d_i$ $(i = 1, 2, \dots, n)$ and the number of dimensions n. Although the value I find for K is not the best possible, the result is sufficient for some applications.

2. *Notations and hypotheses.*

Let Ω be an open bounded set of R^n; we suppose for simplicity $n \geq 3$.

Let us denote by $H^1(\Omega)$ the space obtained by completing $C^1(\bar{\Omega})$ according to the norm

$$(2)\qquad \| u \|_{H^1(\Omega)} = \| u \|_{L_2(\Omega)} + \sum_{i=1}^{n} \| u_{x_i} \|_{L_2(\Omega)} .$$

Let $H_0^1(\Omega)$ be the following subspace of $H^1(\Omega)$:

$$H_0^1(\Omega) = \text{closure of } C_0^1(\Omega) \text{ in } H^1(\Omega).$$

In $H_0^1(\Omega)$ we can assume as a norm the expression

$$(3)\qquad \| u_x \|_{L_2(\Omega)} = \left\{ \sum_{i=1}^{n} \| u_{x_i} \|^2_{L_2(\Omega)} \right\}^{1/2}.$$

In this space the norms (2) and (3) are equivalent, as the following lemma claims.

LEMMA 1. *For any $u \in H_0^1(\Omega)$ it results*

$$\| u \|_{L_{2n/(n-2)}(\Omega)} \leq S \| u_x \|_{L_2(\Omega)}$$

where

$$S = \frac{2(n-1)}{n(n-2)\sqrt{\pi}} \left\{ \frac{n}{2} \Gamma\left(\frac{n}{2} \right) \right\}^{1/n}.$$

PROOF: see e. g. [2] page 488. We observe that the constant S depends only on n and is the best possible.

Then we suppose

$$a_{ij} \in L_\infty(\Omega), \qquad \sum_{i,j=1}^{n} a_{ij} t_i t_j \geq \nu \, | t |^2 \quad \text{a. e. in } \Omega,$$

ν is a positive constant,

$$b_i, \; d_i \in L_n(\Omega), \qquad (i, j = 1, 2, \dots, n),$$

$$c \in L_{n/2}(\Omega), \quad c - \sum_{i=1}^{n} (d_i)_{x_i} \geq 0 \text{ in the sense of distributions,}$$

$$a(u, v) = \int_{\Omega} \left\{ \sum_{i,j=1}^{n} a_{ij} u_{x_i} v_{x_j} + \sum_{i=1}^{n} (b_i u_{x_i} v + d_i u v_{x_i}) + cuv \right\} dx.$$

3. *Main result.*

THEOREM 1. *We suppose that the hypotheses described above are satisfied, and moreover: $f_i \in L_2(\Omega)$ $(i = 1, 2, \dots, n)$, $w \in H_0^1(\Omega)$,*

$$(4)\qquad a(w, v) = \sum_{i=1}^{n} \int_{\Omega} f_i v_{x_i} \, dx \quad \textit{for any} \quad v \in H_0^1(\Omega).$$

Then

$$(5)\qquad \| w_x \|_{L_2(\Omega)} \leq \frac{2^p \sqrt{p}}{\nu} \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega)} \right\}^{1/2}$$

where p is the smallest integer greater than

$$\left\{\frac{2s}{\nu} \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega)}\right\}^n .$$

Proof. The result is trivial if

$$\sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega)} \le \frac{\nu}{2S}$$

(where S is the constant defined in lemma 1).

In fact let us proceed as in [3]: putting $v = w$ in (4) we get

$$\sum_{i=1}^{n} \int_{\Omega} f_i \, w_{x_i} \, dx = a(w, w) = \tag{6}$$

$$= \int_{\Omega} \left\{ \sum_{i,j=1}^{n} a_{ij} \, w_{x_i} \, w_{x_j} + \sum_{i=1}^{n} (b_i + d_i) \, w \, w_{x_i} + c w^2 \right\} dx =$$

$$= \int_{\Omega} \left\{ \sum_{i,j=1}^{n} a_{ij} \, w_{x_i} \, w_{x_j} + \sum_{i=1}^{n} (b_i - d_i) \, w \, w_{x_i} + \left[c - \sum_{i=1}^{n} (d_i)_{x_i} \right] w^2 \right\} dx.$$

Now remembering lemma 1, the hypotheses listed above and Hölder's inequality we get from (6):

$$\frac{\nu}{2} \| w_x \|^2_{L_2(\Omega)} \le \nu \| w_x \|^2_{L_2(\Omega)} - S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega)} \| w_x \|^2_{L_2(\Omega)} \le \tag{7}$$

$$\le \sum_{i=1}^{n} \int_{\Omega} f_i \, w_{x_i} \, dx$$

whence at once

$$\| w_x \|_{L_2(\Omega)} \le 2/\nu \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega)} \right\}^{1/2} .$$

Now let us suppose $\sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega)} > \nu/2S$. Set

$$w_1 = \max (w - k_1 , 0) + \min (w + k_1 , 0)$$

$$\Omega_1 = \left\{ x : \; x \in \Omega, \; \sum_{i=1}^{n} \left| \frac{\partial w_1}{\partial x_i}(x) \right| > 0 \right\}$$

where k_1 is any positive number. Since this function w_1 belongs to $H_0^1(\Omega)$ we can put $v = w_1$ in (4) and we get

$$a(w, w_1) = \int_{\Omega} \left\{ \sum_{i,j=1}^{n} a_{ij} \, w_{x_i} (w_1)_{x_j} + \sum_{i=1}^{n} (b_i - d_i) \, w_{x_i} w_1 + \right. \tag{8}$$

$$\left. + \left[c - \sum_{i=1}^{n} (d_i)_{x_i} \right] w \, w_1 \right\} dx.$$

We observe now that $w w_1 \ge 0$ in Ω, and in the set where $w_1 \neq 0$ it is

$$w_{x_i} = (w_1)_{x_i} \qquad (i = 1, 2, \dots, n).$$

So from (8) it follows

$$a(w, w_1) \ge \int_{\Omega} \left\{ \sum_{i,j=1}^{n} a_{ij} (w_1)_{x_i} (w_1)_{x_j} + \sum_{i=1}^{n} (b_i - d_i) (w_1)_{x_i} w_1 \right\} dx. \tag{9}$$

From (4), (9) we have

$$\nu \| (w_1)_x \|^2_{L_2(\Omega_1)} \le S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_1)} \| (w_1)_x \|^2_{L_2(\Omega_1)} + \tag{10}$$

$$+ \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega_1)} \right\}^{1/2} \| (w_1)_x \|_{L_2(\Omega_1)} .$$

Now it is easy to see that the measure of Ω_1 is a continuous function of k_1, so that $\sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_1)}$ also is a continuous function of k_1. Furthermore it is

$$\lim_{k_1 \to +\infty} \operatorname{meas} \; \Omega_1 = 0. \tag{11}$$

From all these facts it follows that k_1 can be fixed in such a way that

$$S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_1)} = \nu/2. \tag{12}$$

From (10), (12) we get

$$\| (w_1)_x \|_{L_2(\Omega_1)} \le 2/\nu \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega_1)} \right\}^{1/2} . \tag{13}$$

Now suppose $0 \leq k_2 < k_1$ and consider the function

$$w_2 = \min [k_1 - k_2 , \max (w - k_2 , 0)] + \max [k_2 - k_1 , \min (w + k_2 , 0)]$$

and

$$\Omega_2 = \left\{ x : x \in \Omega,\ \sum_{i=1}^{n} \left| \frac{\partial w_2}{\partial x_i}(x) \right| > 0 \right\}.$$

Since w_2 also belongs to $H_0^1(\Omega)$, putting $v = w_2$ in (4) we get

$$(14) \quad \sum_{i=1}^{n} \int_{\Omega_2} f_i (w_2)_{x_i}\, dx = \int_{\Omega} \left\{ \sum_{i,j=1}^{n} a_{ij}\, w_{x_i} (w_2)_{x_j} + \sum_{i=1}^{n} (b_i - d_i)\, w_{x_i} w_2 + \right.$$

$$\left. + \left[c - \sum_{i=1}^{n} (d_i)_{x_i} \right] w\, w_2 \right\} dx .$$

Observe that $w\, w_2 \geq 0$ in Ω, $w_{x_i} w_2 = (w_1 + w_2)_{x_i} w_2$ in Ω,

$$w_{x_i} = (w_2)_{x_i} \text{ in } \Omega_2 \qquad (i = 1, 2, \ldots , n).$$

Therefore from (14) it follows

$$\int_{\Omega_2} \left\{ \sum_{i,j=1}^{n} a_{ij} (w_2)_{x_i} (w_2)_{x_j} + \sum_{i=1}^{n} (b_i - d_i) (w_2)_{x_i} w_2 \right\} dx +$$

$$+ \int_{\Omega_1} \sum_{i=1}^{n} (b_i - d_i) (w_1)_{x_i} w_2\, dx \leq \sum_{i=1}^{n} \int_{\Omega_2} f_i (w_2)_{x_i}\, dx$$

whence, from Hölder's inequality and lemma 1:

$$(15) \quad \| (w_2)_x \|^2_{L_2(\Omega_2)} \leq \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega_2)} \right\}^{1/2} \cdot \| (w_2)_x \|_{L_2(\Omega_2)} +$$

$$+ S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_2)} \cdot \| (w_2)_x \|^2_{L_2(\Omega_2)} +$$

$$+ S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_1)} \cdot \| (w_1)_x \|_{L_2(\Omega_1)}\, \| (w_2)_x \|_{L_2(\Omega_2)} .$$

The measure of Ω_2 is a continuous function of k_2, and it is easy to see that $\lim\limits_{k_2 \to k_1^-} \text{meas } \Omega_2 = 0$. Therefore it is possible to

choose k_2 so that

$$(16) \qquad S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_2)} = \nu/2$$

or $k_2 = 0$ if, with this value, it results

$$S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_2)} \leq \nu/2.$$

From (12), (15), (16) it follows

$$(17) \qquad \| (w_2)_x \|_{L_2(\Omega_2)} \leq 2/\nu \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega_2)} \right\}^{1/2} + \| (w_1)_x \|_{L_2(\Omega_1)} .$$

Continuing the same procedure, let us consider in general some numbers

$$0 = k_p < k_{p-1} < \ldots < k_q < k_{q-1} < \ldots < k_1 .$$

We define

$$w_q = \min [k_{q-1} - k_q , \max (w - k_q , 0)] + \max [k_q - k_{q-1} , \min (w + k_q , 0)]$$

$$\Omega_q = \left\{ x : x \in \Omega,\ \sum_{i=1}^{n} \left| \frac{\partial w_q}{\partial x_i}(x) \right| > 0 \right\},$$

and fix every number k_q such that

$$(18) \qquad \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_q)} = \nu/2S \qquad (q = 1, 2, \ldots , p - 1)$$

$$(19) \qquad \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_p)} \leq \nu/2S.$$

The integer p is therefore determined as the first index for which (19) is valid with $k_p = 0$.

Substituting $v = w_q$ in (4), with easy computations we find

$$\int_{\Omega_q} \left\{ \sum_{i,j=1}^{n} a_{ij} (w_q)_{x_i} (w_q)_{x_j} + \sum_{i=1}^{n} (b_i - d_i) (w_q)_{x_i} w_q \right\} dx +$$

$$+ \int_{\Omega} \left\{ \sum_{i=1}^{n} (b_i - d_i) (w_1 + w_2 + \ldots + w_{q-1})_{x_i} w_q \right\} dx \leq \sum_{i=1}^{n} \int_{\Omega_q} f_i (w_q)_{x_i}\, dx$$

$$(q = 1, 2, \ldots , p).$$

From Schwartz-Hölder's inequality and lemma 1 it follows

$$\nu \| (w_q)_x \|^2_{L_2(\Omega_q)} \leq S \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_q)} \cdot \| (w_q)_x \|^2_{L_2(\Omega_q)} +$$

$$+ \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega_q)} \right\}^{1/2} \cdot \| (w_q)_x \|_{L_2(\Omega_q)} +$$

$$+ S \sum_{j=1}^{q-1} \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_j)} \cdot \| (w_j)_x \|_{L_2(\Omega_j)} \cdot \| (w_q)_x \|_{L_2(\Omega_q)}$$

and from (18), (19) finally

$$\| (w_q)_x \|_{L_2(\Omega_q)} \leq 2/\nu \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega_q)} \right\}^{1/2} + \sum_{j=1}^{q-1} \| (w_j)_x \|_{L_2(\Omega_j)} \qquad (q = 1, 2, \ldots, p). \tag{20}$$

This inequality permits to prove what we want.

In fact it is easy to deduce from (20) that

$$\sum_{q=1}^{p} \| (w_q)_x \|_{L_2(\Omega_q)} \leq \frac{2^p}{\nu} \sum_{q=1}^{p} \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega_q)} \right\}^{1/2}$$

whence, observing that $w_{x_i} = \sum_{q=1}^{p} (w_q)_{x_i}$:

$$\| w_x \|_{L_2(\Omega)} \leq \frac{2^p \sqrt{p}}{\nu} \left\{ \sum_{i=1}^{n} \| f_i \|^2_{L_2(\Omega)} \right\}^{1/2}. \tag{21}$$

With (21) the result is proven if we evaluate p.

From the hypotheses and easy inequalities we get

$$(p-1) \frac{\nu}{2S} = \sum_{q=1}^{p-1} \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega_q)} \leq (p-1)^{1-1/n} \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega)}$$

whence at once

$$p \leq 1 + \left\{ \frac{2S}{\nu} \sum_{i=1}^{n} \| b_i - d_i \|_{L_n(\Omega)} \right\}^n.$$

4. *Example of applications.*

Let us use the preceding result to prove the existence of the solution of a Dirichlet problem in an unbounded domain. Consider an open unbounded set D in R^n, with $R^n - \overline{D} \neq \emptyset$; let V denote the space obtained by completing $C_0^1(D)$ according to the norm

$$\| u_x \|_{L_2(D)} = \left\{ \sum_{i=1}^{n} \| u_{x_i} \|^2_{L_2(D)} \right\}^{1/2}.$$

V is an Hilbert space with the scalar product

$$(u, v)_V = \sum_{i=1}^{n} \int_D u_{x_i} v_{x_i} \, dx.$$

From lemma 1 we get $V \subset L_{2n/(n-2)}(D)$, whence $V \subset L_2^{\mathrm{loc}}(D)$.

Then we suppose $a_{ij} \in L_\infty(D)$, $\sum_{i,j=1}^{n} a_{ij} t_i t_j \geq \nu |t|^2$, ν positive constant, $f_i \in L_2(D)$, b_i, $d_i \in L_n(D)$ $(i = 1, 2, \ldots, n)$, $c \in L_{n/2}(D)$, $c - \sum_{i=1}^{n} (d_i)_{x_i} \geq 0$ in the sense of distributions,

$$a(u, v) = \int_D \left\{ \sum_{i,j=1}^{n} a_{ij} u_{x_i} v_{x_j} + \sum_{i=1}^{n} (b_i u_{x_i} v + d_i u v_{x_i}) + cuv \right\} dx.$$

The result proven in theorem 1 yields the following

COROLLARY 1. *Suppose that the hypotheses mentioned above are satisfied. Then there exists a solution $u \in V$ of the Dirichlet problem*

$$\begin{cases} a(u, v) = \sum_{i=1}^{n} \int_D f_i v_{x_i} \, dx & \forall v \in V, \\ u \in V. \end{cases} \tag{22}$$

PROOF. Let S_m be the ball

$$S_m = \{ x : x \in R^n, \, |x| < m \}$$

and u_m the solution of the Dirichlet problem

$$(23)\qquad \begin{cases} a(u_m, v) = \sum_{i=1}^{n} \int_{D \cap S_m} f_i v_{x_i} \, dx & \forall\, v \in H_0^1(D \cap S_m) \\ u_m \in H_0^1(D \cap S_m) & (m = 1, 2, \ldots). \end{cases}$$

From the hypoteses and theorem 1 we get

$$(24)\qquad \| (u_m)_x \|_{L_2(D \cap S_m)} \leq K \sum_{i=1}^{n} \| f_i \|_{L_2(D \cap S_m)} \quad (m = 1, 2, \ldots)$$

where the constant K depends only on n, ν, $\sum_{i=1}^{n} \| b_i - d_i \|_{L_n(D)}$ and not on m. Since $H_0^1(D \cap S_m) \subset V$ for any $m \in N$, from (24) it follows

$$(25)\qquad \| u_m \|_V \leq K \sum_{i=1}^{n} \| f_i \|_{L_2(D)} \qquad (m = 1, 2, \ldots)$$

From (25) there exists a subsequence, extracted from $\{u_m\}_{m \in N}$, weakly convergent in V to a function u which is a solution of problem (22), as it can easily be verified through (23).

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Alcuni teoremi sull'interpolazione lacunare mediante funzioni polinomiali a tratti

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Summary. — *In this paper we consider a solution of the Turan (0,2) interpolation problem given by piecewise polynomial functions.*

Introduzione.

J. Suranyi e P. Turan hanno studiato, in un loro ben noto articolo [1], il problema dell'esistenza e dell'unicità di un polinomio interpolante, di grado non superiore a $(2n-1)$, che, in n punti assegnati, assume valori assegnati. Negli stessi punti sono dati anche i valori della derivata seconda del polinomio. Per una esauriente bibliografia su tale argomento di interpolazione, detta interpolazione lacunare di tipo (0, 2), si veda, ad esempio, [2].

Nella presente nota viene studiato il problema dell'interpolazione (0, 2) mediante funzioni polinomiali a tratti.

1. Assegnato, nell'intervallo chiuso $I = [a, b]$, il sistema di punti

$$(1.1)\qquad a = x_0 < x_1 < \ldots < x_{N-1} < x_N = b, \qquad N > 3,$$

che dà luogo agli intervalli $I_k = [x_k, x_{k+1}]$, di ampiezza

$$(1.2)\qquad h_{N,k} \equiv h_k = x_{k+1} - x_k, \qquad k = 0, 1, \ldots, N-1,$$

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