UNIVERSITY OF GENOA

Master Degree in Management Engineering

Operations Research 2 – code 60204

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Teacher: Mauro Gaggero

COURSE PROGRAM

1. Introduction and preliminary definitions

1.1. Unconstrained and constrained nonlinear programming problem in standard form
1.2. Definitions of local and global minima and maxima
1.3. Examples of problems that can be formulated as nonlinear programming problems

2. Conditions for the existence of solutions

2.1. Weierstrass theorem and coercive functions

2.2. Quadratic forms and quadratic functions
2.3. Gradient, Hessian, symmetric matrices, positive and negative definite matrices

3. Convex optimization problems

3.1. Definitions and properties of convex functions
3.2. Definitions and properties of convex sets
3.3. Definitions and properties of convex nonlinear programming problems

4. Unconstrained mathematical programming problems

4.1. Necessary optimality conditions and sufficient optimality conditions
4.2. Optimality conditions for quadratic functions
4.3. Usage of optimality conditions for the solution of optimization problems
4.4. Examples of application of optimality conditions for the solution of test problems
4.5. Iterative algorithms and descent algorithms
4.6. Finite and asymptotic convergence property
4.7. Local and global convergence property
4.8. General concepts on descent algorithms: definition of descent direction and descent algorithm
4.9. Discussion of the major issues concerning descent algorithms: choice of the initial point, stop criteria, choice of the descent direction, choice of the descent step
4.10. Common issues of descent methods: local minima trapping and zigzag phenomenon

5. Gradient algorithm

5.1. Definition of the algorithm and principal properties
5.2. Convergence theorem for the steepest descent algorithm with constant descent step
5.3. Definition of rate of convergence of a descent algorithm
5.4. Examples of application of the gradient method to test functions
5.5. Variants of the gradient algorithm: steepest descent algorithm with varying descent step
5.6. Exact line search methods (quadratic and nonquadratic cases) and inexact line search methods (Armijo's rule and decreasing step rule)
5.7. Application of gradient method for the minimization of quadratic functions

6. Newton's algorithm

6.1. Definition of the algorithm and principal properties
6.2. Comparison with the steepest descent method
6.3. Comparison with Newton's method for the solution of equations
6.4. Convergence theorem of Newton's algorithm

6.5. Examples of application of the algorithm to test functions
6.6. Local and global convergence property
6.7. Variants of Newton's algorithm: quasi-Newton's methods and methods for reducing the computational burden

7. Conjugate direction methods

7.1. Introduction to conjugate direction methods
7.2. Definition of conjugate directions
7.3. Conjugate gradient method for quadratic functions
7.4. Finite convergence property of the conjugate gradient algorithm
7.5. Conjugate gradient method for nonquadratic functions: Polak-Ribiere and Fletcher-Reeves algorithms
7.6. Examples of application of the algorithm to test functions

8. Nonderivative methods

8.1. Approximation of the derivatives in descent algorithms with finite differences
8.2. Coordinate descent algorithm
8.3. Powell's algorithm
8.4. Parallel tangent algorithm (Partan)
8.5. Explorative methods based on samplings of the admissible set

9. Line Search methods

9.1. Fibonacci and golden section algorithms
9.2. Methods based on quadratic and cubic interpolations

10. Introduction to constrained nonlinear programming problems

10.1. Optimization problems on convex sets
10.2. Optimality conditions
10.3. Admissible directions algorithms
10.4. Projected gradient algorithm

11. Lagrangian approach for constrained nonlinear programming problems

11.1. Definitions of Lagrangian function, active constraints, and regular constraints
11.2. Necessary optimality conditions and sufficient optimality conditions for equality constraints

11.3. Necessary optimality conditions and sufficient optimality conditions for inequality constraints: Karush-Kuhn-Tucker (KKT) conditions
11.4. Necessary optimality conditions and sufficient optimality conditions for equality and inequality constraints
11.5. Usage of optimality conditions for the solution of optimization problems
11.6. Examples of application of optimality conditions for the solution of test problems

12. Penalty and barrier function methods

12.1. Definition of penalty function methods
12.2. Convergence properties of penalty function methods
12.3. Definition of barrier function methods
12.4. Convergence properties of barrier function methods
12.5. Interior point method for the solution of linear programming problems

References

[1] D. Bertsekas – Nonlinear Programming. Athena Scientific, 1999.
[2] D. Luenberger, Y. Ye – Linear and nonlinear programming. Springer, 2008.