NAME _____ FIRST NAME _____

**Mathematics & O.R.**
**April, 8th 2014**

Part of Geometry and linear algebra. There are two pages in this text. Write the answers on this sheet using the white spaces. Please elucidate every answer in a brief but comprehensive way.

2 points **A**

Find $b$ such that in the $\mathbb{R}$-vector space $C^0[0,b]$ equipped with the usual scalar product (by the integral) the functions $f_1 = x$ and $f_2 = 1 - x^2$ are orthogonal.
Then write the projection of $f_1 + f_2$ onto $L\{f_2\}$.

4 points **B**

Which of the following matrices has the best and which has the worst condition number?

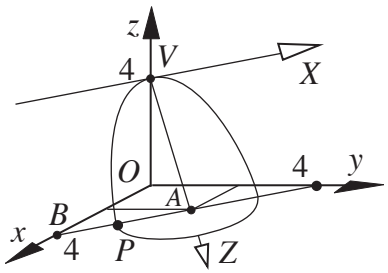
$$A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \end{pmatrix}$$

5 points **C** Let $Q$ be the quadric $x^2 + 4xy - 2y^2 + 8z^2 + k = 0$

1. Find out which kind of quadric is $Q$ for each $k \in \mathbb{R}$
2. For each $k \in \mathbb{R}$ say which conic is the intersection of $Q$ with the plane $z = hy$.
3. Let $k = 0$. Write the tangent plane $\alpha$ to $Q$ in $(0, 2, 1)$ and say which kind of conic is the intersection of $Q$ with $\alpha$.

4 points **D** Let $V(0, 0, 4)$, $A(2, 2, 0)$, $B(4, 0, 0)$, $P(3, 1, 0)$ be four points.

1. Write both direct and inverse formulas for the change of coordinates a change of coordinates such that $V$ i the origin, $Z$ axes passes through $A$ and the $X$ axis is parallel to the line through $B$ and $C$.
2. Find the elliptic paraboloid of revolution, which has $V$ as vertex, the line $VA$ as axis and passes through $P$.



SURNAME _____ FIRST NAME _____

**Mathematics & O.R.**
**April 8th , 2014**

Part of Analysis. There are two pages in this text. Write the answers on this sheet using the white spaces. Please elucidate every answer in a brief but comprehensive way.

4 points **A** Compute the surface integral $\iint_\Sigma f\, d\sigma$ where $f : \mathbb{R}^3 \to \mathbb{R}$ is defined by

$$f(x,y,z) = \frac{x^2}{x^2+y^2}$$

and $\Sigma$ denotes the part of the graph of $g(x,y) = x^2 - y^2 + 3$ which lies in the cylinder $\{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1\}$.

4 points **B** Consider the vector field $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $\mathbf{F}(x,y,z) = (x\cos z\ ,\ y\sin z\ ,\ z^3)$.

Use the divergence theorem to compute the outward flux of $\mathbf{F}$ through the boundary of the cylinder $\{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1 \quad , \quad 0 \leq z \leq 1\}$.

4 points **C** Compute the radius of convergence of the following power series

$$\sum_{n=3}^{+\infty} \frac{n^2}{1+n^3}\, x^n \qquad \sum_{n=0}^{+\infty} \frac{n^2+n!}{n!}\, x^n \qquad \sum_{n=0}^{+\infty} \frac{n!}{1+n^5}\, x^n$$

4 points **D** Consider the $2\pi$-periodic function $f : \mathbb{R} \to \mathbb{R}$, defined by

$$f(t) = t^2 - 2\pi t \quad \text{for} \quad 0 \le t < 2\pi$$

and extended by periodicity to $\mathbb{R}$. Compute the Fourier coefficients of $f$ and write its Fourier series.