NAME ________________ FIRST NAME ________________

Mathematics & O.R.
June, 11th 2014

Part of Geometry and linear algebra. There are two pages in this text. Write the answers on this sheet using the white spaces. Please elucidate every answer in a brief but comprehensive way.

3 points **A** Find $k \in \mathbb{R}$ such that in the $\mathbb{R}$-vector space $C^0[1,2]$ equipped with the usual scalar product (by the integral) the functions $f_1 = x - k$ and $f_2 = 1/x$ are orthogonal. Then find an orthonormal basis for $L\{f_1, f_2\}$

3 points **B** Which of the following matrices has the best and which has the worst condition number?

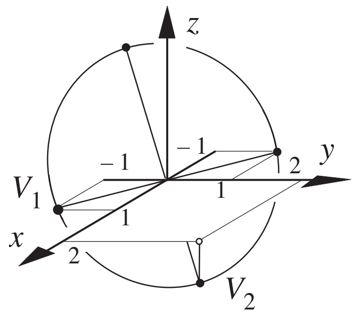
$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} \qquad P = \begin{pmatrix} 2/3 & -1/3 & -2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{pmatrix}$$

5 points **C** Let Q be the quadric $x^2 + y^2 + z^2 + 2xz + 2x + 2z = 0$

1. Find out which kind of quadric is Q.
2. Find the intersections of Q with the line $\{x = t \ : \ y = -t \ ; \ z = 1 + t\}$.
3. Write the tangent plane α in any of the points found above and describe the intersection between Q and α.

5 points **D**

1. Write a cartesian representation for the ellipse with center in $(0, 0, 0)$, and vertices in $V_1(1, -1, 0)$ and $V_2 = (2, 2, -1)$
2. Find the ellipsoid containing the ellipse and passing through $P(1, 1, 1)$.



SURNAME ______________________ FIRST NAME ______________________

Mathematics & O.R.
June 11th, 2014

Part of Analysis. There are two pages in this text. Write the answers on this sheet using the white spaces. Please elucidate every answer in a brief but comprehensive way.

4 points **A** Consider the domain $D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq z \leq 4\}$.

i) Compute the barycenter of D.

ii) Compute the moment of inertia of D with respect to the z axis, that is, the integral

$$\iiint_D (x^2 + y^2)\, dxdydz$$

4 points **B** Consider the surface

$$\Sigma = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4,\ z \geq \sqrt{x^2 + y^2}\}.$$

Compute the surface integral

$$\iint_\Sigma f\, d\sigma,$$

where $f : \mathbb{R}^3 \to \mathbb{R}$ is defined by $f(x, y, z) = e^z + (x^2 + y^2)z$.

4 points **C** i) Compute the radius of convergence of the following power series: $\sum_{n=10}^{+\infty} \frac{n!}{1+e^{2n}n!}\, x^n$.

ii) Determine for which values of the parameter $\alpha \in \mathbb{R}$ the following numerical series converges: $\sum_{n=1}^{+\infty} \frac{(n+\log n)^\alpha}{n^2}$.

4 points **D** Consider the 2π-periodic function $f : \mathbb{R} \to \mathbb{R}$, defined by

$$f(t) = \pi^2 - t^2 \quad \text{for} \quad -\pi \le t < \pi$$

and extended by periodicity to $\mathbb{R}$. Compute the Fourier coefficients of f and write its Fourier series.