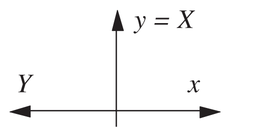
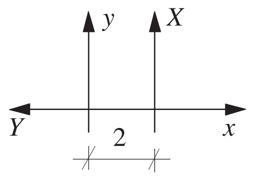
CHANGES OF COORDINATES IN THE PLANE AND SOME CONICS

01. Write direct and inverse formulas for the following changes of coordinates in the plane

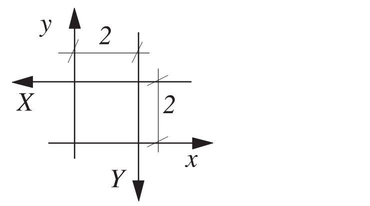
a.



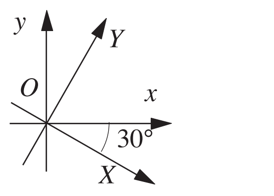
b.



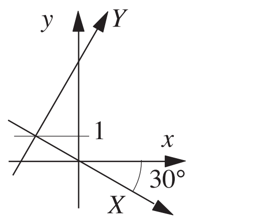
c.



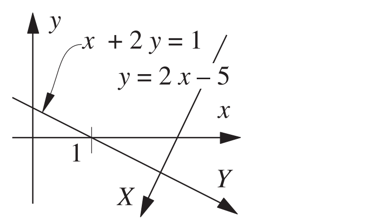
d.



e.



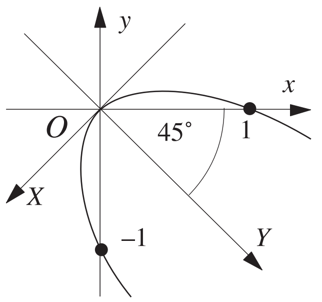
f.



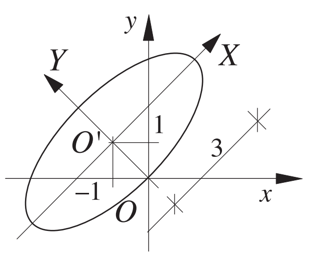
02. Explain why is it possible to construct some system of orthogonal monometric cartesian coordinates in which X axis is the line of equation $12x - 11y + 8 = 0$ (with respect to Oxy) and Y axis is the line of equation $11x + 12y - 10 = 0$. Write direct and inverse formulas for all the possible systems.

03. In each of the following three pictures.
Write direct and inverse formulas for the changes of coordinates between the two systems of coordinates. Then find the equation of the conic with respect to both systems.

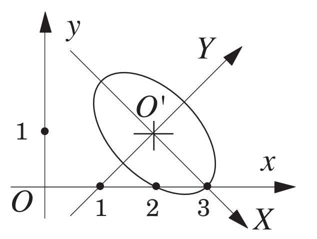
a.



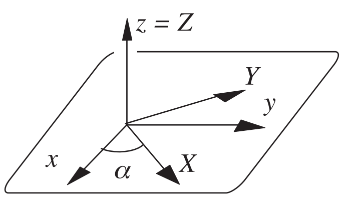
b.



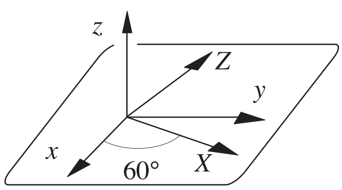
c.



CHANGES OF COORDINATES IN THE SPACE

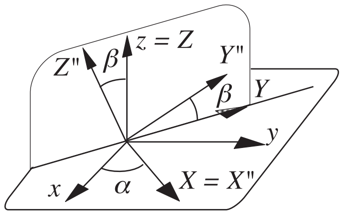


10. Write direct and inverse formulas for the changes of coordinates between system Oxy and system OXY. The second system is constructed by a rotation of α around z axis.

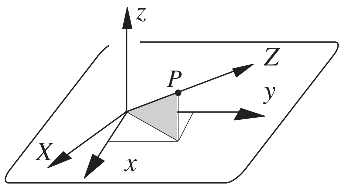


11. In the picture are drawn the system $Oxyz$ and the system $OXYZ$, but Y axis is missing in the picture. Find the direction of Y axis taking care of having two systems with the same orientation, then write direct and inverse formulas for the changes of coordinates between system Oxy and system OXY.

12. Write direct and inverse formulas for the changes of coordinates between system Oxy and system OXY. The second system is constructed by the composition of a rotation of α around z axis and of a rotation of β around X.



13. Write direct and inverse formulas for the changes of coordinates between system Oxy and system OXY. Note that Z axis passes through the point $P\{x = 2, y = 1, z = 1\}$ and X axis lies on the (ground) plane containing x and y axes.



14. Given the two orthogonal lines $\{x = 2y = z\}$, $\{y = 1\ ;\ x + z = k\}$, find a k such that they can be respectively X axis and Y axis of a system $O'XYZ$. In this case find Z axis and the coordinates of O' (with respect to $Oxyz$). Write the changes of coordinates between $Oxyz$ and all possible new systems $O'XYZ$.
15. The two planes $x + 2y - z = 1$ e $x + z = 1$ are orthogonal ans so they can be respectively the planes $[XY]$ and $[YZ]$ of a new system $O'XYZ$. Find all possible coordinates of O' (with respect to $Oxyz$) and the equation of the plane $[XZ]$. Write the changes of coordinates between $Oxyz$ and all possible new systems $O'XYZ$.

CURVES AND SURFACES

20. Let $\mathcal{L}$ be the curve given through a parametric representation $\mathcal{L}\begin{cases} x = \\ y = t - t^2 \\ z = 2t^2 + 1 \end{cases}$
    a. Find the plane containing the curve.
    b. Find the orthogonal projection of $\mathcal{L}$ onto the plane $z = 0$ and onto the plane $x = y$.
    c. Find, if possible, a cartesian representation of $\mathcal{L}$.
21. Let $\mathcal{L}$ be the curve given through a parametric representation $\mathcal{L}\begin{cases} x = t + 1 \\ y = t^2 \\ z = t^3 \end{cases}$.
    a. Prove that there is no plane containing $\mathcal{L}$.
    b. Find the orthogonal projection of $\mathcal{L}$ onto the plane $z = 0$ and onto the plane $x = y$.
    c. Find, if possible, a cartesian representation of $\mathcal{L}$.
    d. Find a cartesian and a parametric representation of the cylinders which contain $\mathcal{L}$ and parallel to z axis, to x axis and to the line $x = 2y = z$.
    e. Find the cones which contain $\mathcal{L}$ and have the vertex in $(0, 0, 0)$ and in $(1, -1, 2)$.
22. Let $\mathcal{L}$ be the curve given through a cartesian representation $\mathcal{L}\begin{cases} x^2 + y - xz = 0 \\ x^2 - y = z \end{cases}$
    a. Find a cartesian and a parametric representation of the cylinders containing $\mathcal{L}$ parallels to x, y, z axes and to the line $\ell : \{2x = y\ ;\ z = 0\}$.
    b. Find, if possible, a cartesian representation of $\mathcal{L}$ as intersection of two cylinders.
30. Find the surfaces of revolution which are the rotation around z axis of the following curves and say which kind of surfaces are they.
    a. The line $\{x = y = z\}$
    b. The line $\{y = 1\ ;\ x = z\}$
    c. The line $\{z = 2\ ;\ y = x + 1\}$
    d. The parabola $\{2z = y^2\ ;\ x = 0\}$
    e. The parabola $\{y = 2z^2\ ;\ x = 1\}$
    f. The circle $\{z^2 + (y - 2)^2 = 1\ ;\ x = 0\}$.
31. Say why the following surfaces are ruled surfaces and write all their lines.
    a. $x^2 - y^2 = xz$
    b. $xy = z$
    c. $x^3y = zx + z$
    d. $x + 3y = z^3$
    e. $(x - z)^2x = y + z$
    f. $x^2 = yz$
    g. $x^2 = y^2z$
32. Find the lines lying on the cone $x^2 - yz = 0$ which intersect the line ℓ, then find the ones which are orthogonal to ℓ. $\ell\begin{cases} x = 1 - t \\ y = 2t \\ z = t \end{cases}$