QUADRICS

01. Simple quadrics that can be identified almost without calculations:

a. $2x^2 - y^2 = 1$ b. $x^2 - y^2 - z^2 = 0$ c. $x^2 - 3y^2 + z^2 - z = 0$
d. $x^2 - 2x + y^2 - z = 0$ e. $x^2 - 2x + y^2 = z^2 - 1$ f. $xy = 1$
g. $x + z^2 + y^2 = 1$ h. $x + z + z^2 = 0$ i. $x + y - x^2 + z = 0$
j. $xy + y^2 = 0$ k. $xy + xz = x$ l. $x^2 - 2xy + y^2 + 1 = 0$

02. a. Identify the quadric $Q: \ x^2 + 5y^2 + z^2 + 5y = 1$.
b. Say why Q is a revolution surface and find the cartesian representation of some circle of radius 1 lying on it.

03. a. Identify the quadric $Q: \ 2xy + 2xz - 4y^2 = 0$.
b. Prove that it is ruled and write all its lines.
c. Find any parabola lying on Q and write its cartesian representation.
d. Describe the intersection between the quadric and the plane $x + y = 1$.

04. a. Identify the quadric $Q: \ x^2 + y^2 + 2xy + 4xz + 4yz - 2x = 0$.
b. Prove that it is ruled and find the two lines of Q passing through $(0, 0, 0)$.

05. a. Say which kind of quadric is $3x^2 + y^2 + 4yz + z^2 + 4x - 2y = 0$
b. Say why it is a surface of revolution and find its axis.
c. Intersect the quadric with the following planes, and identify the resulting conic
$x = y$ tangent plane in $(0, 0, 0)$.
If the intersection is a couple of lines, write a cartesian representation of the lines.

06. a. Say why the quadric $Q: x^2 - 2yz + 2z^2 + 2y - 2z = 0$ is a cone and find its vertex.
b. Write all the lines of the cone.
c. Intersect the cone with the three coordinate planes and in each case say which conic you get.

07. Consider the quadric $x^2 + 2y^2 + 2z^2 + 2yz + 2kz = 0$
a. For each $k \in \mathbb{R}$ identify the quadric.
b. Prove that the quadric is a always a surface of revolution and write its rotation axis.

08. For each $k \in \mathbb{R}$ identify the quadric Q of equation $x^2 + ky^2 - 2xz + 2z^2 + (2k - 2)y = 0$.
Remark: $O(0, 0, 0)$ is always a point of Q. Use the intersection between Q and the tangent plane in O to get some information about Q.

09. a. Identify the quadric $Q: \ x^2 + y^2 + 4xy - z^2 + 2y = 0$.
b. Prove that it is ruled and write all its lines.
c. Prove that Q is a revolution surface and write all its symmetry axes.

10. a. For each $k \in \mathbb{R}$ identify the quadric $2x^2 - y^2 + 4xy + kz^2 + 2y = 0$.
b. Find all the k's for which the quadric is a surface of revolution.
c. Let $k = 1$. Find all the symmetry axes of the quadric and write a change of coordinates such that the quadric gets a canonic equation.

11. a. Prove that the quadric $Q: \ 8xz + 6yz - 2x + 2z = 0$ is a hyperbolic paraboloid.
b. Write all its lines.
c. Find the vertex V (the saddle point) of Q.
d. Find all its symmetry axes.

12. a. Identify the quadric $Q: \ x^2 + 5y^2 + z^2 + 4xy + 6xz + 10yz = 1$
b. Describe the intersection of the quadric with the line $\{x = t + 1 \ ; \ y = t - 1 \ ; \ z = -t\}$.
c. Let P be one of the points of the intersection found above. Describe thoroughly the intersection between the tangent plane to the quadric in P and the quadric itself.

CHANGES OF COORDINATES AND QUADRICS

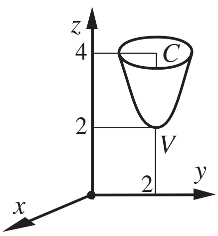
21 Let $V(0,2,2)$ and $C(0,2,4)$ be two points.

Find the equation of the elliptic paraboloid of revolution with the following properties:
- The axis is the line through V and C..
- The vertex is V.
- The circle with center in C and radius 1 lies on it.

22 Find the equation of the hyperboloid of two sheets of revolution such that:
- The vertices are the points $V_1(0,0,4)$ and $V_2(0,0,0)$
- It passes through the point $P(1,1,5)$.

23 Find the equation of the hyperboloid of one sheet of revolution such that:
- The throat circle lies in the plane $y=2$, has center in $C(0,2,0)$ and radius 2.
- It intersects the plane $x=0$ in a circle of radius 3.

21.



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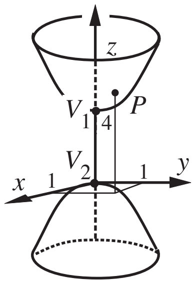

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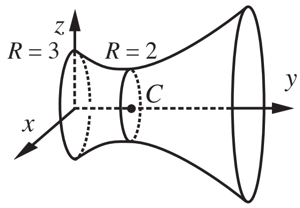
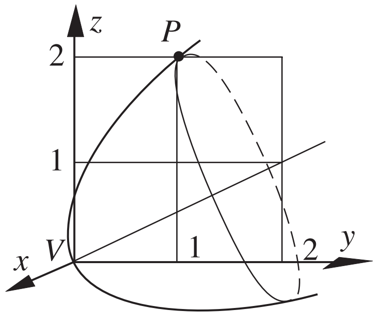


24. a. Find a cartesian representation for the parabola γ with the following features:
   - The vertex is $V(0,0,0)$
   - The axis is the line $\{x=0\ ;\ y=2z\}$
   - Passes through the point $P(0,1,2)$

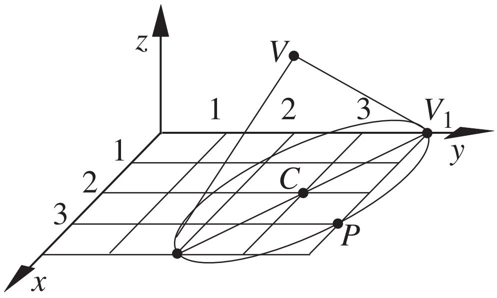
   b. Find the elliptic paraboloid of revolution obtained rotating γ around its axis.



25. a. Find a cartesian representation for the ellipse γ with center in $C(2,3,0)$ that lies on the plane $z=0$, has one vertex in $V_1(0,4,0)$ and passes through the point $P(3,4,0)$.

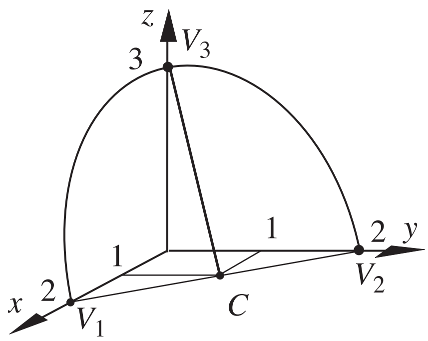
   b. Find the cone with vertex $V(2,3,2)$ that contains γ.

   Hint: The first question is a problem of plane geometry. It is advisable to draw only the plane xy to understand the problem.

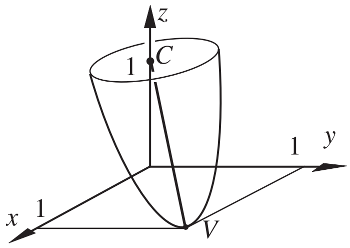


26. a. Find a cartesian representation for the ellipse γ knowing that its center is $C(1,1,0)$ and that the points $V_1(2,0,0)$, $V_2(0,2,0)$, $V_3(0,0,3)$ are three of its vertices.

   b. Find the ellipsoid of revolution obtained by rotating the ellipse around the line $\overline{V_3C}$.



27. Let $V(1,1,0)$, $C(0,0,1)$ be two points and let a be the line through V and C.
Let then γ be the circle γ of radius 1 whose center is C and whose axis is a.
Find a cartesian representation for the elliptic paraboloid which has vertex in V and contains γ.



28. a. Say why the system $\begin{cases} x^2+y^2+z^2+6z=0 \\ x-2y+z=0 \end{cases}$ represents a circle γ and find its center.
b. Find a cartesian representation for the cylinder containing γ and whose generatrices are parallel to the x axis.
c. Find a cartesian representation for the cone of vertex $(0,0,-3)$ containing γ.

29. a. Prove that the three points $C(3,2,3)$, $V_1(2,1,2)$, $C_1(1,0,1)$ are collinear.
b. Find the equation of a hyperboloid of two sheets, with the following features:
- The center is C
- One vertex is V_1
- The circle having center C_1, axis $\overline{CC_1}$ and radius 1 lies on the hyperboloid.

30. Let $A(2,0,\sqrt{2})$, $B(0,2,\sqrt{2})$, $P(1,1,0)$ be three points.
a. Find a cartesian representation for the circle γ passing through the three points.
b. Find the cylinder which contains γ and has generatrices parallel to x axis.
c. Find the cone which contains γ and has vertex $V(0,0,\sqrt{2})$.