Option pricing via Radial Basis Functions: Performance comparison with traditional numerical integration scheme and parameters choice for a reliable pricing

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Abstract

In order to price options with an equity underlying, Financial Institutions often value them using Black-Scholes framework. Starting from six hypothesis of economic nature and assuming that the asset, which the option is based on, follows a geometric brownian dynamic (SDE), the authors defined the well-known Black-Scholes-Merton fundamental Partial Differential Equation (BSM PDE).

The role of BSM PDE is to describe the most likely future dynamic of the option underlying, whereas the financial instruments characterization depends on the derivative pay-off and it is realized through the specification of the initial conditions (IC) and the Dirichlets boundary conditions (BC).

For standard contracts, called plain-vanilla derivatives, and for a few class of non-standard instruments, called exotic derivatives, this problem can be solved analytically reaching a theoretical fair value through a closed formula (CF) valuation, otherwise a numerical method must be used.

Classical numerical integration schemes, which have been implemented for this purpose, are Finite Difference Method (FDM) and Finite Elements Method (FEM).

In addiction to these techniques, other mathematical approaches can be used in quantitative finance, such as Monte Carlo methods and Markov Chains, also known as stochastic trees (CRR Cox Ross Rubistein Tree).

Other things being equal, these methods always converge to the same fair-value, even if with different performances, according to the pricing problem to be solved.

In the last ten years, financial engineering has focused on an innovative methodology for option pricing which has its foundations on Radial Basis Functions (RBF).

This technique was conceived independently by Hardy and Duchon as a multidimensional interpolation method in 1970 and it subsequently was considered a predecessor of the modern Neural Network (NN).

In 1990, Kansa reviewed the operational steps in order to solve elliptical, parabolic and hyperbolic PDE. His new point of view was successful and many problems of traditional engineering and physics were solved by applying Radial Basis Functions.

These days, RBF are becoming more and more popular in the field of quantitative finance for different reasons:

1) These are methods designed for working in high-dimensional spaces, that means options involving more underlying.

2) Given that RBF works in a similar way to NN, this technique has an accurate fitting

3) RBF is a mesh-free method, therefore the algorithmic implementation is easier than traditional scheme (FEM) in a multi-dimensional space.

4) The computed functions, as well as their derivatives, are smoother than those ones obtained by traditional integration scheme. This is a very interesting feature for the estimation of option greeks, that is the sensitivity of the fair-value varying according to financial parameters.

5) Scientific literature shows that RBF method is able to solve all the three PDE canonical forms in many different technical applications.

Among these, the feasibility to apply RBF on the parabolic PDE form is a very interesting feature because the BMS PDE, through a variables modification, can be transformed into a heat-equation.
However this new approach has also some drawbacks connected to the possibility of creating ill-conditioned matrices and thus they cannot be processed through the time integration engine.

This problem is strictly connected to the number of collocation points used for solving PDE, to the chosen radial basis function and to its shape parameter.

This paper aims to examine how this technique works in the financial field, to compare the RBF performance with the results obtained with traditional methods (FDM, FEM), to choose the more suitable radial basis function to solve option pricing and to explain how its shape parameters can be set.

This last aspect must be considered one of the more critical issue and it is the object of numerous debates in the scientific community.

It is crucial to set properly the shape parameter for the precision of the method and ultimately for the determination of the derivatives fair-value.

Applying a Maximum Likelihood Estimation (MLE), the authors propose a financial approach for its evaluation based on market/theorical prices calibration.

**Key words:** Radial Basis Functions (RBF), Shape parameter calibration, Meshless method in Quantitative Finance, Certificate and Option Pricing, Partial Differential Equation (PDE)

**REFERENCES**